INVERSE BOUNDARY VALUE PROBLEMS OF AERОHYDRODYNAMICS BY MEANS OF THE METHODS OF NUMERICAL OPTIMIZATION

A. M. Elizarov and E. V. Fedorov UDC 533.69:532.5+519.6+517.5

As is well known, in inverse boundary value problems of the theory of analytic functions two boundary conditions are specified on unknown portions of the boundary [1]. If one of the latter is replaced by an optimized portion, the inverse problem is called a variational problem [2]. Variational inverse boundary value problems of aerohydrodynamics (IBPA) may consist in the construction of wing profiles possessing optimized characteristics (maximum lift, minimum resistance, et al.). Statements of such problems go back essentially to a paper of Lavrent'ev [3] in which a formulation was given for the problem of optimizing the shape of a profile, taking into account conditions guaranteeing nonseparation of the flow over a large portion of its contour.

It follows from the results given in [4] (see [5]) that among profiles with a smooth contour a circle provides maximum lift under streamline flow. However, it does not conform to the requirements of aerodynamic design if only because, under a real flow, separation of the fluid from its surface occurs.

A numerical-analytical method for maximizing the lift of a profile with a sharp outside edge, flown over by an ideal incompressible fluid, was proposed in [5] under restrictions expressing simplified conditions for nonseparation of the flow. The magnitude of the lift R_y was expressed in terms of the Fourier coefficients of a function connected with the conformal mapping of the unit disk in an auxiliary plane onto the profile exterior. An extremum of R_y is attained by varying a finite set of these coefficients under indicated additional restrictions. Conditions for closure of the contour are satisfied empirically.

Solutions are also known of variational IBPA, which take into account viscosity of the flow in the framework of boundary layer theory. In a number of papers (for example, [6-9]) profile resistance or aerodynamic quality was optimized for a family of profiles with a parametrically specified contour, flown over by a viscous incompressible fluid under various assumptions concerning structure of the boundary layer. As constraints, the lift coefficient C_y, the angle of attack α, and the maximum thickness of the profile, or α and the profile area, were specified, as well as a criterion for nonseparation of the flow. For each profile of such a family, as a result of solving the direct problem aerodynamic characteristics were calculated and their optimization was carried out by selecting values of the free parameters under the constraints indicated above. Analogous results were obtained in problems involving subsonic or transonic flow of a gas, possibly viscous, around profiles. A survey of these papers can be found in [10].

A second approach to aerodynamic optimization of wing profiles, foundations for which are presented in [11], is based on the theory of inverse boundary value problems [1] and is developed in the present paper. In what follows, optimized profiles, flown over without separation, are constructed under various nonseparability conditions, a comparison of these solutions is given, and conclusions are made. It is shown that the nonseparability condition for a complete turbulent boundary layer ensures nonseparability of the flow even in the presence of laminar cells on a mixed part of the contour. Conditions are presented for physical realizability of a solution, these being imposed on a set of control functions and connected with the construction of single-sheeted wing profiles. Optimization problems are also studied with additional constraints on aerodynamic characteristics for a range of attack angles.

1. Functionals for Optimization of Aerodynamic Characteristics. The following integral representation of the solution of a basic IBPA [1] was used in [11] for optimization of the shape of wing profiles:

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$$z(\zeta)=e^{-i\beta}\int\limits_{1}^{\zeta}\exp\left[-(2\pi)^{-1}\int\limits_{0}^{2\pi}g(\gamma)\frac{e^{i\gamma}+\zeta}{e^{i\gamma}-\zeta}d\gamma\right]d\zeta,\quad |\zeta|>1. \tag{1.1}$$

Here $g(\gamma)=a_0+P(\gamma)+h(\gamma)$; $h(\gamma)=(\varepsilon-1)\ln[2\sin(\gamma/2)]$; a_0, ε, and β are constants ($1\leqslant\varepsilon\leqslant 2$, $0\leqslant\beta\leqslant\pi/2$); the Hölder control function $P(\gamma)$ satisfies the conditions

$$\int\limits_{0}^{2\pi}P(\gamma)\,e^{i\gamma}d\gamma=\pi(\varepsilon-1),\quad \int\limits_{0}^{2\pi}P(\gamma)\,d\gamma=0. \tag{1.2}$$

In Eq. (1.1) function $z(\zeta)$ maps the exterior of the unit circle in the ζ-plane conformally onto the exterior of the wing profile, bounded by a closed piecewise-Lyapunov contour L_z with a sharp edge at the point $z = 0$, where the angle interior to the flow domain is equal to $\varepsilon\pi$, and flown over by an ideal incompressible fluid at a given incident flow rate. The constant β is equal to the theoretical angle of attack (angle of inclination of the profile to the direction of irrotational flow), and the constant a_0 determines the magnitude L of the perimeter L_z.

We consider a class of profiles with a fixed perimeter L of the contour. Let us assume that each of the profiles is flown over by a smooth plane-parallel unbounded established flow of an incompressible viscous fluid at substantively large Reynolds numbers. We select a system of coordinates such that the axis of abscissas is parallel to the velocity vector of the oncoming flow; the magnitude v_∞ of this velocity and the fluid density ρ are assumed to be known. In modeling such a flow, with smallness of the displacement thickness δ^* of the boundary layer and wake taken into account, potential flow around a profile by an ideal fluid has been considered approximately (see, for example, [12, 13]) as displacements of a potential flow around a semi-infinite body. Under the assumptions indicated, the lift R_y, the profile drag R_x, and the aerodynamic quality K may be expressed, taking relation (1.1) into account, in the form of the following functionals [11] specified on a set of admissible functions $P(\gamma)$:

$$R_y=2^{3-\varepsilon}\pi\rho v_\infty^2 L\sin\beta/J(P),\quad R_x=2^{2,2}\rho v_\infty^2 L\,(2Aa)^{1/a}\,\mathrm{Re}^{1/a-1}D(P),$$
$$K=R_y/R_x=2^{-1,2}\pi\sin\beta\,(2Aa)^{-1/a}\,\mathrm{Re}^{1-1/a}/E(P). \tag{1.3}$$

Here

$$J(P)=\int\limits_{0}^{2\pi}[\sin(\gamma/2)]^{\varepsilon-1}\exp P(\gamma)\,d\gamma;$$

$$D(P)=B(P)\,[J(P)]^{-1/a};\ E(P)=B(P)\,[J(P)]^{1-1/a};$$

$$B(P)=\left\{\left[\int\limits_{0}^{\pi+2\beta}G_2(P;\gamma)\,d\gamma\right]^{1/a}+\left[\int\limits_{\pi+2\beta}^{2\pi}G_2(P;\gamma)\,d\gamma\right]^{1/a}\right\}\{\cos\beta\exp[-P(0)]\}^r,\quad r=3,2-(7-a)/(3-a);$$

$\mathrm{Re} = v_\infty L/\nu$; A and a are interrelated empirical constants determined by the method for calculating the turbulent boundary layer (specific values for them are given below); ν is the known kinematic coefficient of viscosity;

$$G_2(P;\gamma)=\sin(\gamma/2)\,|\cos(\gamma/2-\beta)|^{3(a+1)/(3-a)}\exp[-4aP(\gamma)/(3-a)]. \tag{1.4}$$

We note that for known models for calculation of the boundary layer the quantity r differs insignificantly from zero. This makes it possible to simplify somewhat the expression for B(P), putting $r = 0$.

Thus, by virtue of the functions (1.3), for fixed ρ, v_∞, and β, to maximize R_y and K and to minimize R_x it is necessary to minimize the corresponding functionals J(P), E(P), and D(P) on the set U of control functions $P(\gamma)$ which, besides the conditions (1.2), must satisfy the condition for nonseparability of the flow and guarantee the physical realizability of the resulting solution.

2. Constraints on the Control Functions. As remarked above, one of the basic constraints on the control functions $P(\gamma)$ must be the condition of nonseparability of flow over the corresponding profile. In the approximation of boundary layer theory known criteria for the absence of separation of a turbulent flow can be written in the form [12, 13]

$$f(s)\geqslant f_*,\quad f_*=-\mu/A\quad (f(s)=v'(s)\,f_t(s)); \tag{2.1}$$

$$f_t(s) = a\,|v(s)|^{-b}\left|\int_{s_t}^{s}[v(\tau)]^{b-1}d\tau + C\right|, \tag{2.2}$$

where $a = (m + 1)/m$; $b = 2(4m + 1)/(2m - 1)$; $v(s)$ is the velocity distributing along L_z; s $(0 \leqslant s \leqslant L)$ is the arcwise abscissa, reckoned from the trailing edge, so that the flow domain stays to the left; s_t, $(s_* \leqslant s_t < L$ or $0 < s_t \leqslant s_*)$ is the abscissa of the point of transition from laminar to turbulent flow on the upper or lower surfaces of L_z; s_* is the arcwise abscissa of the branch point of the flow; and f_*, A, and m are known empirical constants. In particular, $m = 4$, $-5.57 \lesssim f_* \lesssim -4.77$, according to Prandtl–Burr; $m = 6$, $-3 \lesssim f_* \lesssim -2$, according to Loitsyanskii; $m = \infty$, $-0.8 \lesssim f_* \lesssim -0.7$, according to Bam–Zelikovich; $m = 6$, $f_* = -6$, according to Kochin–Loitsyanskii; $A = 0.01256$ for $m = 4$, and $A = 0.00655$ for $m = 6$ [12, Sec. 54]. Constant C in relation (2.2) characterizes the laminar flow portion and can be calculated, for example, from the formula [14, Sec. 12]

$$C = v_t^{b-2}\nu\,(\mathrm{Re}_t^{**})^{a}/(aA), \quad \mathrm{Re}_t^{**} = \left[a_1\nu^{-1}v_t^{2-b_1}\left|\int_{s_*}^{s_t}[v(\tau)]^{b_1-1}d\tau\right|\right]^{1/2} \tag{2.3}$$

$[a_1 = 0.45,\ b_1 = 5.35,\ v_t = v(s_t)]$. To determine the location of the boundary layer transition point s_t, we employ an empirical criterion, due to Euler [15], according to which there is a transition from laminar to turbulent flow if

$$\ln \mathrm{Re}^{**}(s) \geqslant \mathrm{Re}_T = 18{,}4H_{32} - 21{,}74,\ \mathrm{Re}^{**}(s) = v(s)\,\delta^{**}(s)/\nu,$$

where $H_{32} = H_{32}(s)$ is the depth of energy loss in the boundary layer; $\delta^{**}(s)$ is the depth of momentum loss. Applying the method for calculation of the laminar layer [14, Sec. 112], we determine specific values of Re_T by interpolating with respect to the quantity H, which is, in turn, a function of the form-parameter f (see Table 17 in [14]). As a result of the interpolation, we obtain the monotonically decreasing function

$$H_{32} = G(H),\ 1{,}9 \leqslant H \leqslant 4{,}03,\ 1{,}515 = G(4{,}03) \leqslant G(H) \leqslant G(1{,}9) = 1{,}7. \tag{2.4}$$

Thus, the quantity s_t must be the smallest of the roots of the equation

$$\mathrm{Re}^{**}(s) = \exp\{18{,}4G[H(s)] - 21{,}74\}. \tag{2.5}$$

We assume that point S_t is located on the mixed portion. Using this, we show that the following estimate applies:

$$f_t(s) \leqslant f_0(s). \tag{2.6}$$

Here $f_0(s)$ is given by expression (2.2) when $s_t = s_*$, $C = 0$, and there is a purely turbulent boundary layer.

Let s correspond to points of the upper surface of the profile (the case of the lower surface is considered in a similar way). By virtue of relations (2.2), (2.3), to establish inequality (2.6) it is sufficient to justify the estimate

$$C \leqslant \int_{s_*}^{s_t}[v(\tau)]^{b-1}d\tau.$$

Noting that $b_1 > b$ and $v(s)/v_t \leqslant 1$ for $s_* \leqslant s \leqslant s_t$, we have

$$\int_{s_*}^{s_t}[v(\tau)]^{b-1}d\tau = v_t^{b-1}\int_{s_*}^{s_t}[v(\tau)/v_t]^{b-1}d\tau \geqslant v_t^{b-b_1}\int_{s_*}^{s_t}[v(\tau)]^{b_1-1}d\tau.$$

From relations (2.3) it follows that

$$\int_{s_*}^{s_t}[v(\tau)]^{b_1-1}d\tau = \nu\,\mathrm{Re}_t^{**2}v_t^{b_1-2}/a_1.$$

Using the preceding inequality, we derive the result

$$\int_{s_*}^{s_t}[v(\tau)]^{b-1}d\tau \geqslant \nu\,\mathrm{Re}_t^{**2}v_t^{b_1-2}/a_1.$$

Hence, inequality (2.6) will be satisfied if $\mathrm{Re}_t^{**2-a} \geqslant a_1/(aA)$. It follows from relations (2.4) and (2.5) that $\mathrm{Re}_t^{**a} \geqslant (462.2)^{2-a}$. Thus, for the satisfaction of relation (2.6) it is sufficient that parameters a and A satisfy the constraint $a_1/(aA) \leqslant (462.2)^{2-a}$. Verification of the last inequality for various sets of parameters a and A has shown that this inequality is valid. We have thus established the property (2.6). It follows from this that to guarantee non-separation of the flow for the case in which s_t is located on the mixed portion, it is sufficient to require fulfillment of condition (2.1), assuming a completely turbulent boundary layer ($s_t = s_*$, C = 0). The latter condition, in the expression involving function $P(\gamma)$ with $\varepsilon = 2$, takes on the form

$$(-1)^j G_1(P;\gamma) \geqslant f_{0j} G_0(P;\gamma), \quad f_{0j} \geqslant f_*, \quad j = 1, 2,$$
$$\gamma \in [0, \pi+2\beta] \text{ for } j=1, \ \gamma \in [\pi+2\beta, 2\pi] \text{ for } j=2,$$
$$G_0(P;\gamma) = G_2(P;\gamma)\left[a\left|\int\limits_{\pi+2\beta}^{\gamma} G_2(P;\gamma)\,d\gamma\right|\right]^{-1}, \tag{2.7}$$
$$G_1(P;\gamma) = -P'(\gamma) - 0{,}5\,\mathrm{tg}(\gamma/2-\beta),$$

operator G_2 is defined in relation (1.4); j = 1 corresponding to the upper surface of L_z and j = 2 corresponds to the lower surface. Since inequality (2.1) can hold everywhere on L_z only when $\varepsilon = 2$, throughout the sequel it is this case we consider. We note that functional J(P) is strictly convex, while the function $P_*(\gamma) = (1-\varepsilon)\ln(2\sin(\gamma/2))$, yielding its global minimum in space L_2 under conditions (1.2), is not Hölderian and does not satisfy relation (2.7). Consequently, the extremum of J(P) is attained on the boundary of a set of Hölderian functions satisfying conditions (1.2) and (2.7). The situation is similar in problems involving minimization of E(P) and D(P).

3. Conditions for Physical Realizability of a Solution. The constraints (1.2) and (2.7) on the set U do not guarantee physical realizability of a solution; in particular, a profile with a simple contour. Difficulties in guaranteeing this property are connected with the absence of necessary and sufficient conditions for single-sheetedness of a solution, which would be expressed in terms of the function $P(\gamma)$. Simplicity of contour L_z can be achieved if on U we impose an arbitrary one of the sufficient conditions for single-sheetedness of an IBPA solution (see, for example, [16]). However, these conditions characterize only certain subclasses of the set of single-sheeted solutions, which leads to a subtantial contraction of the set U. It is therefore more expedient to contract U at the expense of eliminating from it part of the non-one-sheeted solutions by using the necessary conditions for one-sheetedness. We indicate some of these, expressing them rather simply in terms of the function $P(\gamma)$.

We consider a set of profiles with an infinitely thin trailing edge ($\varepsilon = 2$), bounded by Lyapunov contours L_z, the curvature of which is everywhere bounded. By developing a proposition of G. Yu. Stepanov concerning the impossibility for single-sheeted profiles of distributions v(s) increasing monotonically from a branch point to a point of descent of the flow, presented at a symposium in Irkutsk in 1988 on "Modern Problems of the Mechanics of Fluids and Gases," F. G. Avkhadiev showed (in a report at a seminar at the Scientific-Research Institute of Mathematics and Mechanics of Kazan University, Nov. No. 14, 1989) that the following condition is sufficient for non-single-sheetedness of the flow domain:†

$$v_{01}(\gamma) + v_{02}(2\pi-\gamma) \leqslant 2v_0(0), \quad 0 \leqslant \gamma \leqslant \pi. \tag{3.1}$$

Here $v_0(\gamma) = |v_0[s(\gamma)]|$; $v_0(s)$ is the velocity distribution on L_z corresponding to the irrotational flow mode; $v_{01}(\gamma) = v_0(\gamma)$ for $0 \leqslant \gamma \leqslant \pi$ (on the upper surface of L_z); $v_{02}(\gamma) = v_0(\gamma)$ for $\pi \leqslant \gamma \leqslant 2\pi$ (on the lower surface). It follows naturally from relation (3.1) that for single-sheeted profiles monotonically increasing distributions $v_0(s)$ are inadmissible.

It follows from F. G. Avkhadiev's proposition that for simple closed L_z satisfaction of the inequality opposite to inequality (3.1) is necessary:

$$\max_{0 \leqslant \gamma \leqslant \pi} [v_{01}(\gamma) + v_{02}(2\pi-\gamma)] > 2v_0(0). \tag{3.2}$$

Using the condition

$$v[s(\gamma)] = 2u_0|\cos(\gamma/2-\beta)|\exp[-a_0 - P(\gamma)], \tag{3.3}$$

where u_0 is a known constant, we derive the following inequality from inequality (3.2):

†The result is mentioned with the consent of the author.

$$\max_{0\leqslant\gamma\leqslant\pi} \Phi(P;\gamma)>0,\quad \Phi(P;\ \gamma)=\cos(\gamma/2)\{\exp[-P(\gamma)]+\exp[-P(2\pi-\gamma)]\}-2\exp[-P(0)]. \qquad (3.4)$$

When inequality (3.4) is satisfied, we also have the inequality

$$\max_{0\leqslant\gamma\leqslant 2\pi} \{|\cos(\gamma/2)|\exp[-P(\gamma)]\}>\exp[-P(0)], \qquad (3.5)$$

equivalent to the condition $\max\limits_{0\leqslant\gamma\leqslant 2\pi} v_0(\gamma)>v_0(0)$. It, by virtue of condition (3.1), is also a necessary condition for single-sheetedness; however, it determines a set of admissible functions $P(\gamma)$ much broader than (3.4). We remark that in the case of symmetric profiles the conditions (3.4) and (3.5) coincide.

Numerical calculations have shown, with minimization of the functionals E(P), D(P) for arbitrary β and minimization of J(P) for $\beta \geqslant 0.2$, that not taking into account additional requirements connected with a guarantee of physical realizability of a solution leads to a non-single-sheeted profiles, and, among the optimal profiles obtained using condition (3.4), one does encounter non-single-sheeted profiles. It is therefore convenient to apply, instead of inequality (3.4), more stringent constraints. One of them, constructed by analogy with inequality (3.2) and used in a computational experiment, may be written as follows:

$$v_{01}(\gamma)+v_{02}(2\pi-\gamma)>2v_0(0),\ 0\leqslant\gamma\leqslant\gamma_0<\pi \qquad (3.6)$$

(γ_0 is a fixed quantity). In an expression involving $P(\gamma)$ inequality (3.6) takes the form

$$\Phi(P;\ \gamma)>0,\ 0\leqslant\gamma\leqslant\gamma_0. \qquad (3.7)$$

Being more stringent than relation (3.4), the constraint (3.7) allows for the possibility of a loss of single-sheeted solutions. On the other hand, for values of γ_0 close to π, taking the constraint (3.7) into account did, as calculations have shown, yield single-sheeted solutions.

4. Numerical Optimization. Functions $P(\gamma)$, providing a minimum to the functionals J, D, and E on the set U, were sought in the form of trigonometric polynomials $P_N(\gamma)=\sum\limits_{l=1}^{N}(a_l\cos l\gamma+b_l\sin l\gamma)$, where N is a given quantity and, by virtue of conditions (1.2), $a_1 = \varepsilon - 1$, $b_1 = 0$. In order to guarantee that $P_N(\gamma)$ possess the Hölder property, with fixed coefficient A_0 the following constraints were imposed on the constants a_ℓ, b_ℓ:

$$|a_l|\leqslant(A_0-1)/[2^{1/2}l(N-1)],\ |b_l|\leqslant(A_0-1)/[2^{1/2}l(N-1)],\ l=2,\ \ldots,\ N,$$

where $A_0 \approx 10^2$. Calculations have shown that such a value for A_0 has no effect on the optimization results when there is a substantial increase in the length N of the segment of Fourier series, i.e., the solution obtained is automatically Hölderian with the indicated coefficient. To numerically realize the conditions for nonseparation on the interval [0, 2π], points γ_j, $j = 1,\ldots,M_1$ were selected, at each of which satisfaction of the corresponding inequality in relations (2.7) was verified. In a similar way, points $\hat{\gamma}_j$, $j = 1,\ldots,M_2$, were selected in order to implement the inequality (3.6) on the interval $[0, \gamma_0]$.

Considering the functionals being minimized as functions of 2(N − 1) independent coefficients a_ℓ, b_ℓ, we obtain a nonlinear programming problem of dimensionality $2(N-1)(M_1+M_2)$. Its solution was accomplished by the relaxation method presented in [17]; this required substantial expenditure of machine time in connection with the calculation of the integrals appearing in the functionals being minimized and the function $G_0(P;\ \gamma)$. In particular, for N = 5, $M_1 = 27$, $M_2 = 13$, one iteration requires 40 sec for minimization of D(P) and 30 sec for minimization of J(P), with processing time on the ES-1046 computer. Thus N proves to have the largest influence on the expenditure of machine time. For example, in minimizing J(P), using the same values for M_1 and M_2, a two-fold increase in N led to an increase of up to 140 sec of computer time for one iteration. The total number of iterations oscillated from units to several tens of iterations depending on the choice of initial approximation.

Figures 1-4 present optimized profiles, nonseparated according to Loitsyanskii. The choice of this criterion may be explained by the fact that it gives the best agreement, in comparison with other criteria, of results on nonseparability in calculations with respect to more precise models.

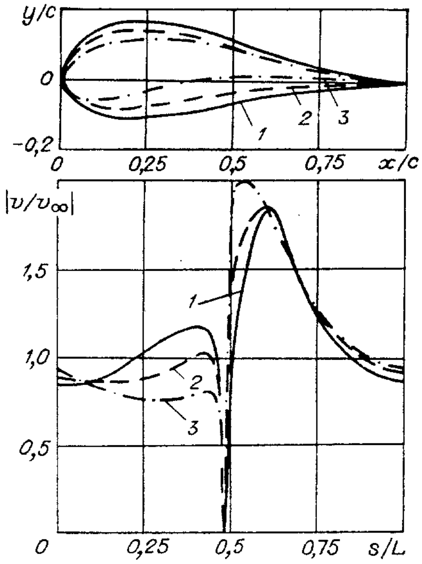

Fig. 1

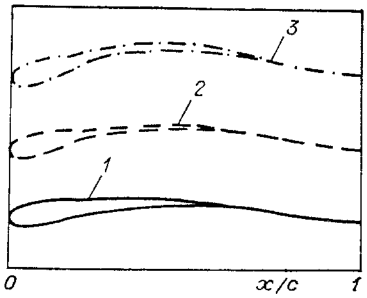

Fig. 2

Figure 1 shows the maximum lift profiles [and the v(s) distributions for them] obtained for various β: (β = 0.1 rad = 5.7°, continuous curve; β = 0.15, dashed curve; β = 0.2, dash–dot curve). The arc-abscissa is referred to the perimeter L; velocities are referred to the given value v_∞, and contour coordinates are referred to the chord c. Profiles 1-3 in Fig. 1 have lift coefficients C_y = 0.748, 1.084, 1.388, respectively, for angles of attack α = 3.72, 5.8, 8.64°. It is evident that with an increase in β (hence also in α) profiles, optimal in the sense of lift, become thinner and C_y increases. Thus, an increase in the lift takes place mainly at the expense of a change in v(s) on the lower surface of the profile. There is an essential increase in the velocity in a neighborhood of a branch point of the flow.

Figure 2 presents single-sheeted profiles 1-3 of minimum resistance, obtained for the same values of β for Re = 10^6 and have C_x = 0.0122, 0.0133, and 0.0155 for α = 2, 3.4, and 5.4°.

The profiles described are, first of all, of theoretical interest. However, possessing the corresponding extremal properties, they can, like Liebeck's profile [18], serve as distinctive guide lines defining the direction for seeking optimal aerodynamic forms.

Further, the calculations have shown that for fixed values of β the functionals E(P) and D(P) differ little from one another in their behavior. This explains the agreement in the profiles of maximum aerodynamic quality and minimum resistance, obtained as a result of minimizing the indicated functionals (Fig. 2, K = 53.8, 81.7, and 84.2).

If the value of β is not fixed beforehand, then it, along with the coefficients a_ℓ, b_ℓ are among the parameters being sought for optimization. It follows from relations (1.3) that to maximize R_y it is necessary in this case to minimize the functional $J_0(P;\ \beta) = J(P)/\sin\beta$, where $P \in U$, $0 \leqslant \beta \leqslant \pi/2$. The computational experiment has shown that with an increase in β the minimal values of J(P) also increase, but much more slowly than $\sin\beta$. There arises, therefore, the problem of finding the maximum value of β for which the set of admissible solutions is nonempty. It has been shown that as β decreases the minimum value of the functional $D_0(P;\ \beta)$, expressing profile resistance, decreases and the corresponding profile becomes thinner, approximating a plate in form, flown over at zero angle of attack, and also, as N increases the optimal profiles become thinner.

Thus, with β variable, to the maximization of R_y there corresponds an increase in angle β; to the minimization of R_x there corresponds a decrease in β to zero. As for the functional $E_0(P;\ \beta) = E(P)/\sin\beta$, expressing aerodynamic quality, as β increases the minimum value of E_0 first decreases and then increases. There is, as a result, a unique value of β corresponding to the absolute minimum of E_0.

Figure 3 displays profiles 1-3, obtained as a result of minimizing J_0, D_0, and E_0, respectively. Profile 1 maximizes R_y and has C_y = 1.499 with attack angle α = 9.6°. Profile 2 minimizes R_x and has relative thickness 0.037, C_x = 0.0101 for α = 0 and Re = 10^6. We

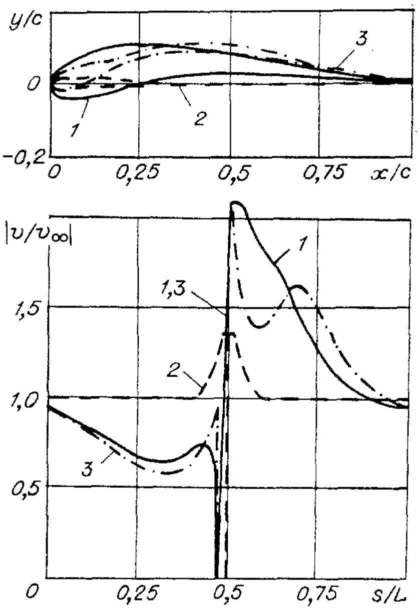

Fig. 3

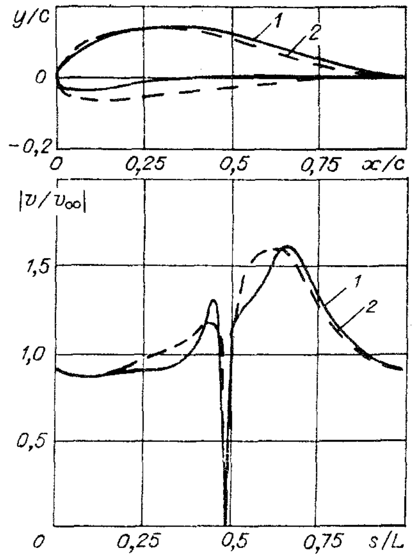

Fig. 4

note that the resistance coefficient found differs little from the theoretical value of C_x for a plate (see, for example, [14]), which for Re = 10^6 for a purely turbulent boundary layer is equal to 0.0094. Profile 3 maximizes K and has C_y = 1.382, C_x = 0.0163 for α = 6° and Re = 10^6. For this profile K = 84.7. For comparison, note that for profile 1 we have C_x = 0.0199 and K = 75.3 for the same Re.

5. Problems with Additional Constraints on Aerodynamic Characteristics. Along with the problems described above, a case of interest is that in which one of the aerodynamic characteristics is optimized with constraints on the others [for example, on coefficients C_y, C_x or with a maximum value for v(s) on the contour]. By virtue of relations (1.3), all of these constraints can be expressed directly in terms of $P(\gamma)$.

Let $C_x \leqslant \widehat{C}_x$ ($\widehat{C}_x$ is a given quantity). Taking note of relations (2.9) and the fact that for real profiles the chord, as a rule, amounts to 47-49% of the contour perimeter, the indicated constraint on C_x can, for fixed β, be replaced, with an accuracy of 1-2%, by the constraint

$$D(P) \leqslant \widehat{D} = \widehat{C}_x [2^{3,2}(2.4a)^{1/n} \mathrm{Re}^{-1/(m+1)} d]^{-1}, \; d = 2{,}04 - 2{,}13. \tag{5.1}$$

Inequality (5.1) serves as an additional constraint on the set U in the minimization of J(P).

In Fig. 4 curve 1 represents the solution of this problem for β = 0.1, Re = 10^6 and $\hat{C}_x$ = 0.015. For the profile we have C_y = 0.706 for α = 0.78°.

Assume now that the additional constraint $C_y \geqslant \widehat{C}_y$ is specified. By virtue of relations (2.8), for fixed β this is equivalent to the inequality

$$J(P) \leqslant \widehat{J} = 4\pi d \sin\beta / \widehat{C}_y. \tag{5.2}$$

The problem obtained is that of minizing the functional D(P) on set U with the additional constraint (5.2). In Fig. 4, profile 2 is the solution of the latter problem for β = 0.1, Re = 10^6, and C_y = 0.720; here C_x = 0.0159 and α = 2.6°.

We remark that the problem described in the present section may be easily extended to the case in which the quantity β is variable.

6. Optimization with Nonseparation Conditions Taken into Account for a Range of Angles of Attack. As is well known, even for small changes in the angle of attack due to separation arising in the flow, significant deterioration in the aerodynamic characteristics of a wing profile can occur. It is therefore important, from the practical point of view, to seek optimized profiles with nonseparation of the flow over them over a fairly broad range of angles of attack.

In [19] a condition was obtained for the absence of separation of a completely turbulent boundary layer for a change in the angle of attack α over a given range $\delta = \alpha_1 - \alpha_2$, $\alpha_2 \leqslant \alpha \leqslant \alpha_1$. This condition has the form

$$f(v_j;\, s_j,\, s) \geqslant f_*, \quad s \in [s_1, L] \text{ for } j = 1, \quad s \in [0, s_2] \text{ for } j = 2. \tag{6.1}$$

Here $v_j(s)$ are the velocity distributions on the upper ($j = 1$) and lower ($j = 2$) profile surfaces for flow over them at attack angles α_1 and α_2, respectively; s_j are arc-abscissas of branch points of the flow for $\alpha = \alpha_j$. The form-parameter $f(v_j;\, s_j,\, s)$ is given by the formula (2.2) for $C = 0$, $s_t = s_j$ and $v(s) \equiv v_j(s)$. Changing over in relation (6.1) from $v_j(s)$ to $v_j(\gamma)$ [see relation (3.3)] and noting that to the abscissas s_j there correspond the polar angles $\gamma_j = \pi + 2\beta_j$ ($\beta_2 \leqslant \beta \leqslant \beta_1$) on the unit circle, we obtain the condition for non-separation of the flow over the profile in the range $\delta = \alpha_1 - \alpha_2 = \beta_1 - \beta_2$ in the form (2.7), where $\beta = \beta_1$ for $j = 1$ and $\beta = \beta_2$ for $j = 2$. The named condition is easily used in place of condition (2.7) as a constraint on the set of control functions $P(\gamma)$.

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